

# Sierpinski Triangle

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  - Sierpinski
  - Fractal
  
- 2 Properties of Sierpinski Triangle
  - Dimension
  - 3D Sierpinski triangle

# Table of Contents

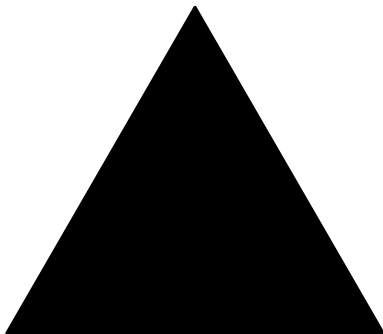
## 1 Definition of Sierpinski Triangle

- Sierpinski
- Fractal

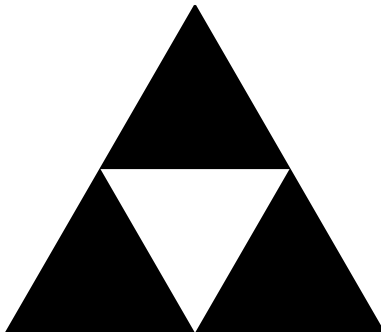
## 2 Properties of Sierpinski Triangle

- Dimension
- 3D Sierpinski triangle

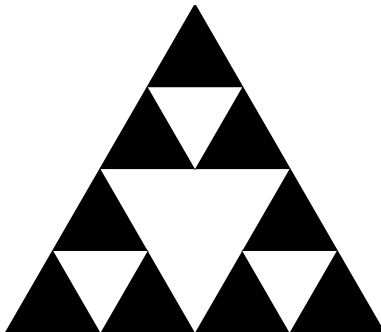
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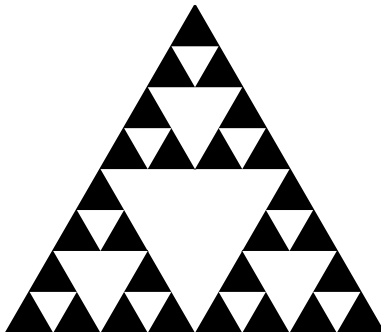
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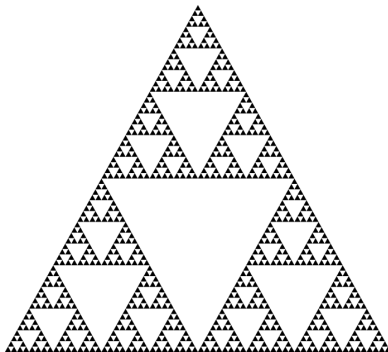
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# Sierpinski Triangle

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→ **HARD TO READ!!!**



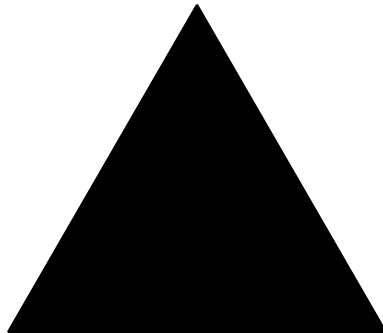
# Sierpinski Triangle

- How to Draw Sierpinski Triangle

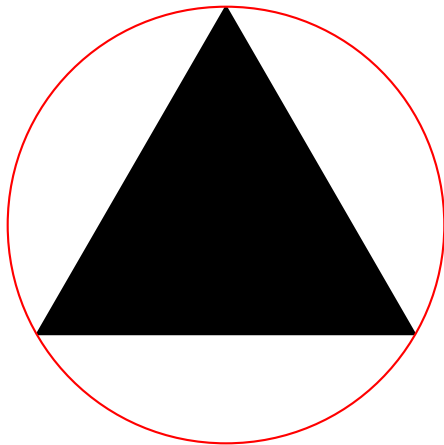
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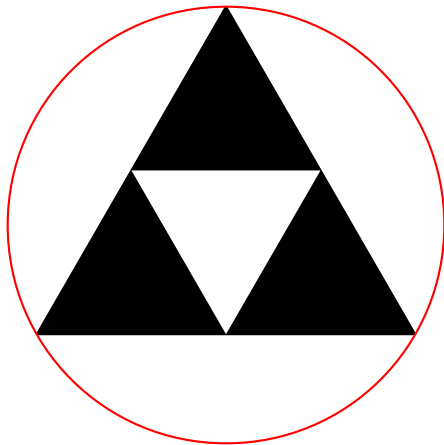
# Sierpinski Triangle



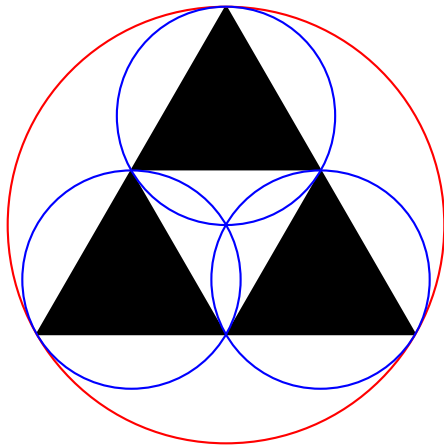
# Sierpinski Triangle



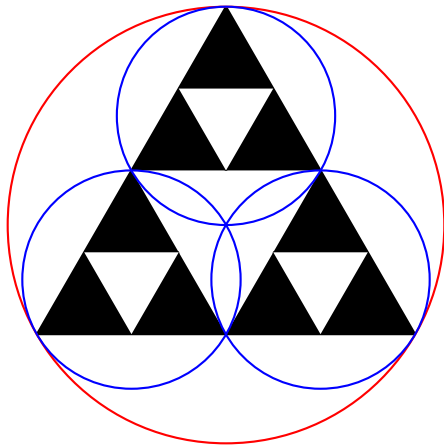
# Sierpinski Triangle



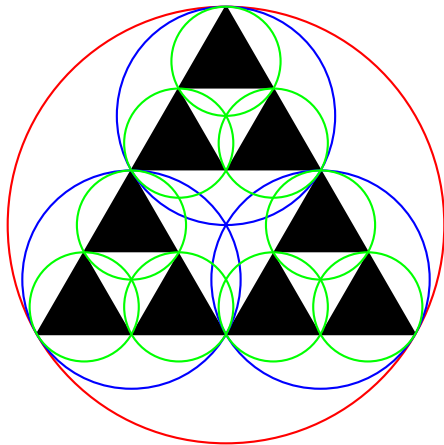
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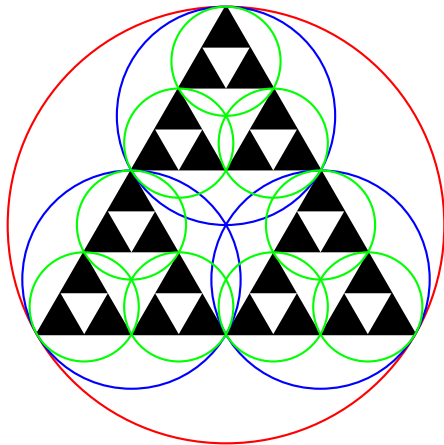
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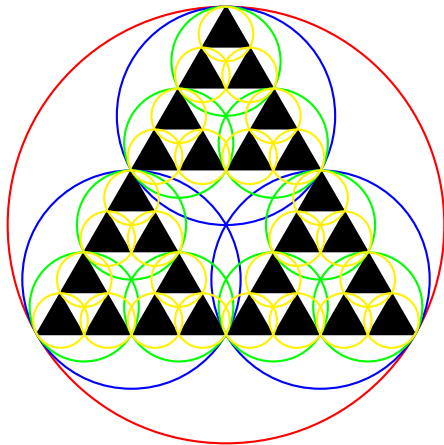


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- Then, what's fractal?

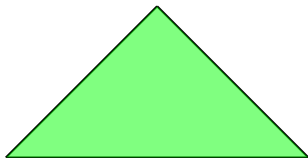
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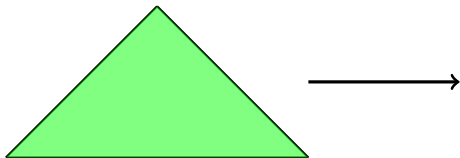
- Category?
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- Then, what's fractal? Something that has...
  - Self-similarity
  - Recursive Definition

# Sierpinski Triangle : Self-Similarity

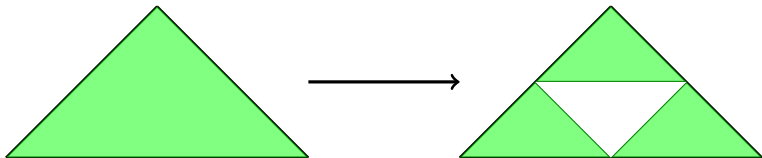




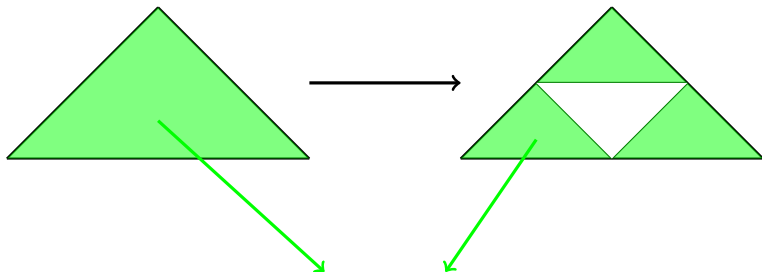
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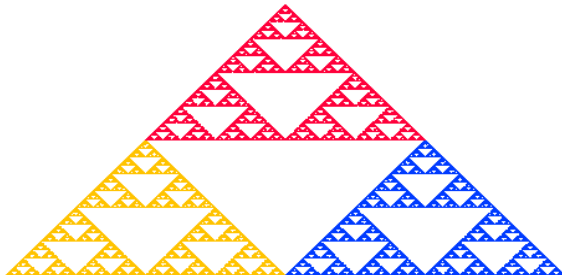


# Sierpinski Triangle : Self-Similarity

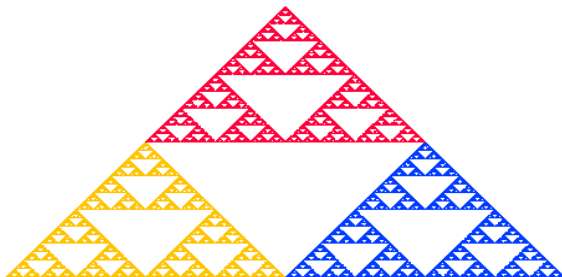


Same Shape!

# Sierpinski Triangle : Recursive Definition



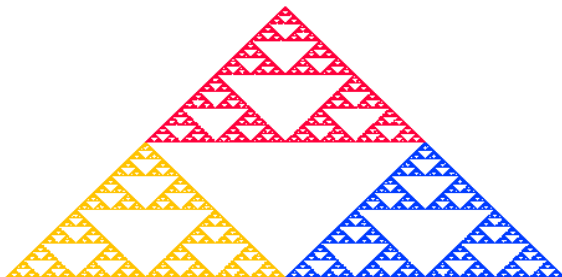
# Sierpinski Triangle : Recursive Definition



Yellow :

$$x_{n+1} = 0.5x_n + 0.25 \quad y_{n+1} = 0.5x_n + 0.5\frac{\sqrt{3}}{2} \quad (1)$$

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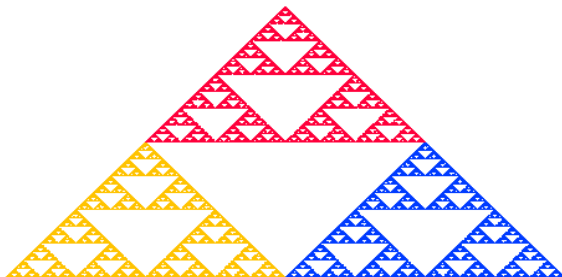
Yellow :

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Blue :

$$x_{n+1} = 0.5x_n + 0.5 \quad y_{n+1} = 0.5x_n \quad (3)$$

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# Dimension



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We know that a line has a dimension of 1

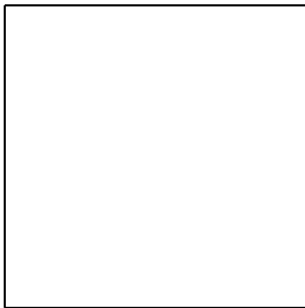
# Dimension

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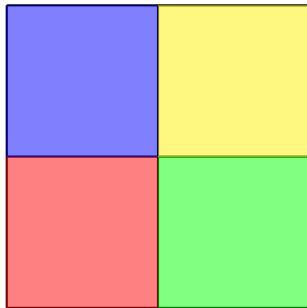
We know that a line has a dimension of 1

We can divide it into two identical parts

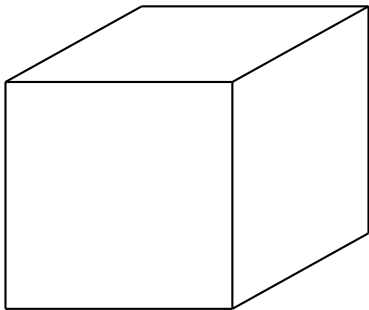
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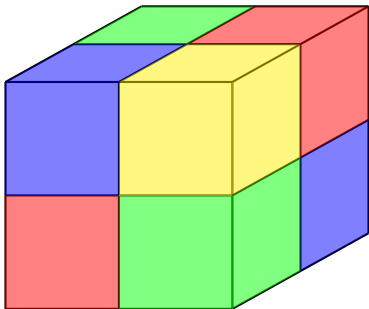
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The fractals are made up of points not lines or surfaces.

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The fractals are made up of **POINTS** not lines or surfaces.

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⇒ I'm using lines and surfaces to help your understanding.

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- line : 2 identical lines

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So what?!

# Dimension

Let's find the relationship between the scale and the number of similar shapes.



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- We know that dimension of a cube is 3

$$\frac{1}{\left(\frac{1}{2}\right)^3} = 8 \quad (6)$$

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$$\frac{1}{S^D} = N \quad (7)$$

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S : Scale, D : Dimension, N : Number of Identical Shape

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$$D = -\frac{\log N}{\log S} \quad (8)$$



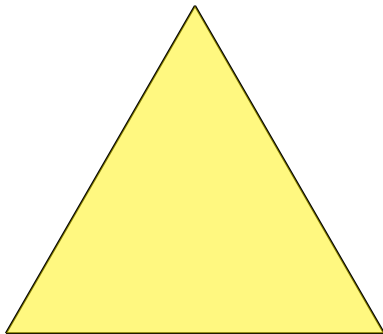
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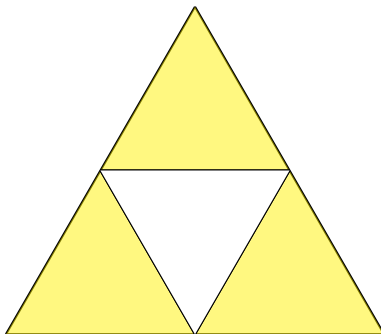
$$D = -\frac{\log N}{\log S} \quad (8)$$

Now, we can get the dimension of the Sierpinski Triangle.

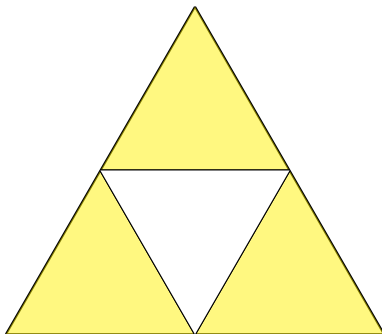
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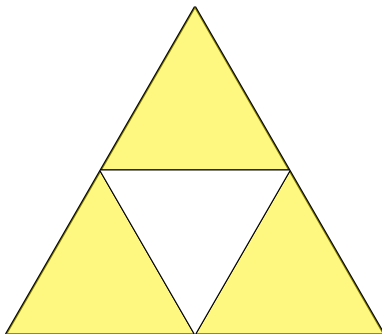


# Dimension



Division Scale =  $\frac{1}{2}$ , Number of Identical Shapes = 3

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$$A_{final} \rightarrow 0 \quad L \rightarrow \infty$$

# Dimension

We figured that,

$$D_{\text{Sierpinski}}$$

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$$\approx 1.58$$

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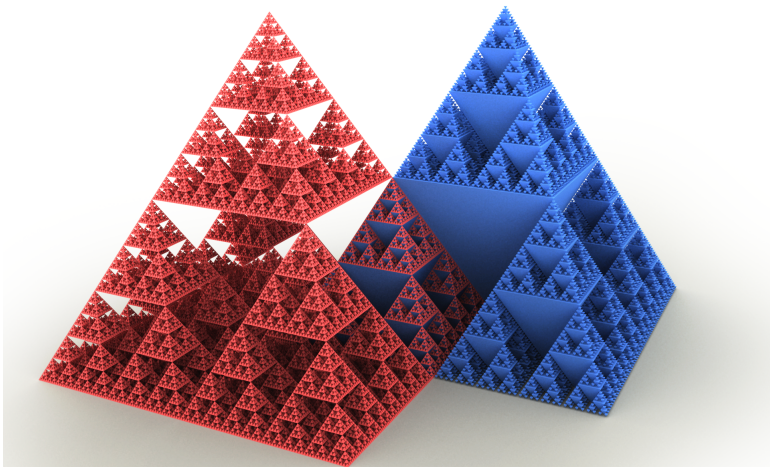
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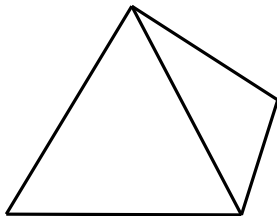
$$\approx 1.58 \quad (11)$$

How about on 3D?

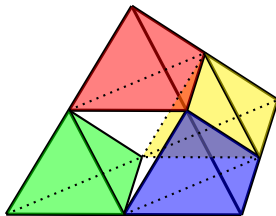
# 3D Sierpinski Triangle



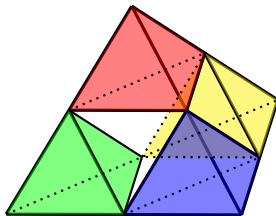
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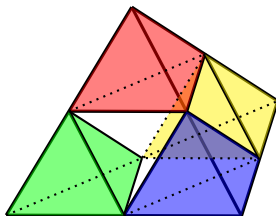


# 3D Sierpinski Triangle



Division Scale =  $\frac{1}{2}$ , Number of Identical Shapes = 4

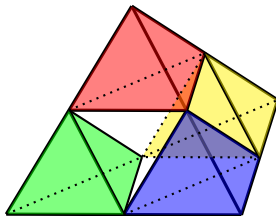
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$$A_{initial} = A_{final} \quad V \rightarrow 0$$

# 3D Sierpinski Triangle



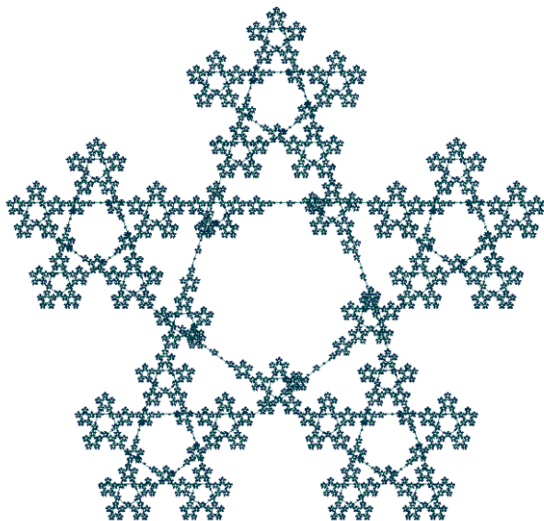
Division Scale =  $\frac{1}{2}$ , Number of Identical Shapes = 4

$$A_{initial} = A_{final} \quad V \rightarrow 0$$

$$D_{3D \text{ Sierpinski}} = -\frac{\log 4}{\log \left(\frac{1}{2}\right)} = 2$$




# 3D Sierpinski Triangle

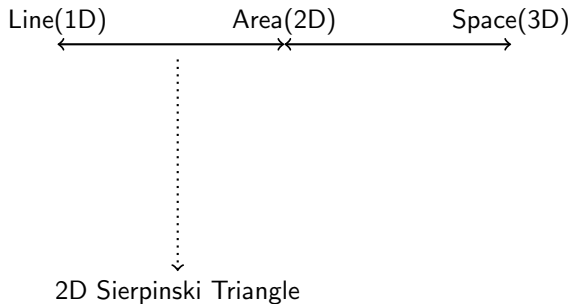


# Conclusion

Line(1D)                      Area(2D)                      Space(3D)



# Conclusion



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